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Experimental mathematics practices and mathematical modeling in math teachers in training for the construction of proposals for new ways of teaching quadratic function

Prácticas de matemática experimental y modelación matemática en profesores de matemática en formación para la construcción de propuestas de nuevas formas de enseñanza de la función cuadrática

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Abstract:
The present investigation arose from observing traditional strategies used by math teachers, therefore, it was sought that math teachers in training build new ways of teaching the quadratic function, based on theories of the experimental mathematics practices and mathematical modeling . The methodology is phased: design experiment and modeling eliciting activities (MEA). As significant findings, the strategies used by the teachers in training and the identification of innovative proposals to teach math in the classroom were characterized.
key words: experimental mathematics, mathematical modeling, design experiments, model eliciting activities (MEA).

Resumen:
La presente investigación surgió al observar estrategias tradicionales utilizadas por los profesores de matemática, por tanto, se buscó que profesores de matemática en formación, construyan nuevas formas de enseñanza de la función cuadrática, basándose en las teorías de la práctica de matemática experimental y modelación matemática. La metodología es por fases: design experiment y modeling eliciting activities (MEA). Como hallazgos significativos se caracterizó las estrategias utilizadas por los profesores en formación y la identificación de propuestas innovadoras para enseñar de la matemática en el aula.
Palabras clave: matemática experimental, modelación matemática, experimentos de enseñanza (design experiment), actividades reveladoras del pensamiento (MEA).

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1. Introduction

Mathematics, more than an area of study, is a tool that helps man understand his reality, in which to understand the magnitude of his contributions in other disciplines and in everyday life, requires a change in the conception of this, because it has been established in people's thinking, as a complex discipline and difficult to approach, so it is necessary to build new strategies for their permanent education, which are chords and coherent with the constant situation of experimentation and change in which we live in (Hernández, 2014).

On the other hand, there is currently an interest in studying the math teachers in training, their pedagogical practices and the impact they have on the processes of teaching and learning of mathematics (Valbuena, Conde, Padilla, 2018), for therefore, this research in the current context can contribute significant elements in this area.

In addition, although the quadratic function is a central concept in the process of learning mathematics, its understanding remains a problematic issue for students, since they are not attracted to traditional teaching strategies (Conde-Carmona and Fontalvo, 2019), due to the traditional way of approaching the study of the quadratic function, which is through different constructions of tables of values and graphical representations on paper, which not only present a lot of inaccuracy, but also facilitate less the processes of transfer and application. For his part, Villa (2009) states that in order for the student to reach better levels in terms of mathematical thinking, he must develop mathematics activities in the teaching-learning process centered on the mathematical modeling processes, which allows predictions to be made, taken decisions, since, this is interrelated and complemented with the formulation (which can be given through experimental mathematics) and solving problems of everyday life.

Therefore, teachers have the need to seek new teaching strategies for quadratic function. So Ponte (2012) states: “The teaching of quality mathematics necessarily goes through a teacher with an appropriate mathematical training, with competence in the didactic field, with a good relationship with students, a professional attitude and their ability to professional performance”. Currently, the challenge of teacher education is to design strategies that facilitate the use of constructivist methods for teaching mathematics in all grade levels, that is, preparing teachers to teach mathematics for their understanding and application and not their memorization.

Making a review of the literature, the benefits that have the experimental mathematics practices and mathematical modeling in the teaching of any subject in this area, were found, which is to build a concept loaded with meaning, based on the observation of a phenomenon of the real context and construction of a model of this phenomenon, which will generate significant learning at the end. But to have a correct application of these practices in the classroom it is needed a prepared and convinced teacher for such action. On the other hand, Niss (2007), states: it is essential to incorporate the experimental mathematics practices and mathematical modeling into the curriculum of teachers in training for the development of teaching competences related to these practices, such as for the establishment of environments and the approach In modeling situations, however, researches have shown that the vast majority of educational programs for teachers in training do not provide knowledge and experiences that cause teachers to use these practices in the classroom (Alsina, 2007; Conde-Carmona, 2019).

Due to the above, the following question arises: What impact do the use of experimental mathematics practices and mathematical modeling have on math teachers in training for the construction of proposals for new ways of teaching quadratic function?

1.1. Experimental mathematics

According to Pérez (1992), he states that experimental mathematics is usually associated with the following characteristics:

- They are based on the scientific method of trial and error.
- Mathematization occurs through a model building process.
- The emphasis on deductive demonstrations is diminished and more attention is given to the contributions of evidence and conjecture, thus connecting with the informal math line of Lakatos.
- Determine a “natural” space for learning: the workshop or the laboratory.

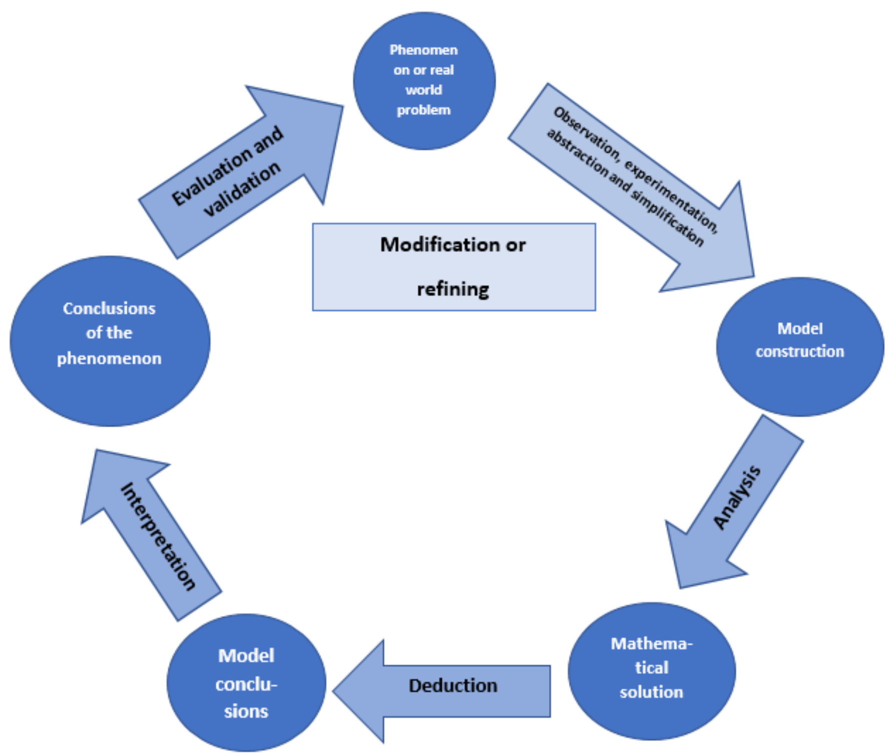
Therefore, experimental mathematics, its methods and techniques developed allow a teaching of a specific mathematical theme, but with the use of resources that enable the subject's action and its role in learning.

1.2. Mathematical modeling

Mathematical modeling should be understood as an activity that is not exhausted in the production of mathematical representations articulated to the study situation, but also it recognizes other aspects of human nature and the role of mathematics in society. Therefore, the mathematical modeling and the contexts from which it emerges, have different intentions and play different roles within the classroom, also implies a series of actions or phases that make the construction or interpretation of a model not carried out instant way in the classroom, these actions or phases are known as the modeling cycle (Villa, 2009).

Therefore, Villa (2009) proposes a cycle of mathematical modeling, comprised of several phases as shown in the following figure:

Figure 1
Mathematical modeling processes



Source: taken from (Villa, 2009)

As can be seen in the figure, the cycle begins with the determination of a phenomenon or problem of the "real world", which is observed and subjected to a process of experimentation with what is intended to deepen their understanding and the search for data. Then, simplifications and assumptions that eliminate some of these are made, to build a model that represents the phenomenon. Once the model is built, all possible analyzes are generated and mathematical tools are used to construct a theoretical solution from which the conclusions of the model are derived; these conclusions must be subsequently interpreted in the light of the phenomenon. In the search for coherence between the conclusions of the model and the phenomenon itself, evaluation and validation strategies are proposed. If, after validation, the model is consistent with the problem phenomenon, the cycle ends; otherwise, it begins again based on the evaluation of the phenomenon enriched with the analyzes, an observation is made, the data, the variables are adjusted and the modification of the model is continued.

2. Methodology

The methodology used in this research, is based on the performance of teaching experiments, proposed by Cobb and Steffe, who highlight that this methodology allows the researcher to design learning environments for a better understanding of its elements (student, mathematical object, context, artifact or technological resource, situation-problem, structured activity, etc.), as well as anticipating how these will work together to promote learning (Villareal, 2003).

Methodologically, this work has two phases, the first one is the design of the experiment, in this case, the researchers will design an activity (session one) based on the experimental mathematics practices and mathematical modeling so that the math teachers in training can relate to this, and the second one, model eliciting activities (MEA) and it is used in this research because its intended is that math teachers in training build, based on the activity carried out in session one, a problem solving scheme with mathematical modeling and also, build new forms of teaching quadratic function based on the experimental mathematics practices and mathematical modeling, carried out in session #1 and #2.

Two characteristics are presented in the design experiment, that make this methodology relevant and focused on this research, one of the characteristics is that allows us to assume the role of researchers and teachers, in this way you can vary and make decisions in the experiment formulated specifically in the free fall experiment proposed by the researchers. The second one is intervention, in this case, it is intended to guide math teachers in training at the time of conducting the experiment proposed by the researchers.

On the other hand, the MEA (model eliciting activities), is also known as “activities that generate models” or “activities to obtain structures”, according to Cobb and Steffe (1983), this phase must involve interesting mathematical systems that evidence the student the need to build or formulate models for the interpretation and explanation of the situation.

The revealing activities of thought evoke meaningful learning, since they relate new ideas to aspects of cognitive structure. For these authors, the fundamental thing in solving a problem that involves revealing thinking activities is to express, test and revise models that allow solving the problem.

In this space, the revealing activities of thought will be worked in groups of 2 to 3 members in the second and third session of the calculus teaching class at the Universidad del Atlántico, in the second session, the math teachers will be asked in training to build a problem solving scheme with mathematical modeling taking into account the experience they had when carrying out the activity proposed by the researchers in the first session. In the third session, the math teachers in training, taking into account the experience they had when carrying out the activities of session one and two, propose a new way of teaching the quadratic function with experimental mathematics practices and mathematical modeling as a final product, following the order of ideas of the objectives set in the research work.

3. Results

3.1. Design experiment analysis

When carrying out session one, certain difficulties were present in the math teachers in training, the most consistent were:

The use of tenths and hundredths of a second in the sexagesimal system: this part became an epistemological obstacle, which are psychological difficulties that do not allow a right appropriation of objective knowledge (Bachelard, 1981). In relation to what has been mentioned above, the math teachers in training took the time that the ball lasted from falling from one floor to another with their respective height, but that time taken represented it in tenths and hundredths in the sexagesimal system being that these are given in the decimal system. This use is evidenced in the graph when they locate the data collected in the Cartesian plane.

The misuse of scales in the Cartesian plane: this difficulty was the most persistent in the math teachers in training at the time of graphing the data, since they did not use a proportional scale. It can be evidenced that the math teachers in training located the data taken from the experiment on different scales, that is, it did not take a proportional scale, which led to the graph of the data was not correct, and therefore It was not a parable, but a straight line. What makes it difficult to recognize the behavior of the data and the real identity of the function that will allow to build the model.

Inappropriate use of graphs for data analysis: they performed data analysis through a diagram, which should be a scatter diagram. Instead to analyze the data of the experiment, they went to make other types of diagrams, in a timely case, they made a bar chart.

Strong tendency to use rule of three without taking into account its application to linearity: in the activity process they were asked to show a mathematical model that would allow them to develop each item and help them find a mathematical equation that models the proposed phenomenon by the researcher. The math teachers in training used a rule of three, to partially solve the proposed problem, but doing the step-by-step activity, they realized that this mathematical model did not fit the behavior of the experiment data.

3.2. Model eliciting activities analysis

Respecting to the analysis of the activities that generate models, the proposal in session 2 was analyzed, in which the math teacher in training is asked to construct a problem-solving scheme with mathematical modeling where they will show, in general, what they did in the activity of session # 1. It should be noted that this scheme should be a tool to solve any mathematical modeling problem that may arise. In this regard, the following table shows a contrast of the mathematical modeling process proposed by Villa (2009) and the process presented by math teachers in training in their scheme:

Table 1
Scheme analysis

Phases presented by Villa (2009) \ Modeling scheme by teachers in training	Group #1	Group #2	Group #3	Group #4
Phase 1: Real problem phenomenon (exploration, experimentation, abstraction and simplification)	✓	✓	✓	X
Phase 2: Model construction (analysis)	✓	✓	X	✓
Phase 3: Mathematical solution (deduction)	X	✓	✓	✓
Phase 4: Model conclusion (interpretation)	X	✓	✓	✓
Phase 5: Conclusion of the phenomenon (evaluation and validation)	✓	✓	✓	X

Source: self made

Group # 1: In the problem solving scheme with mathematical modeling proposed by this group, a first phase was evidenced, which an observation of the experiment and exploration of the data found was made. Next, they indicate the performance of the analysis of the data collected and then be represented by a graph, this is reflected in the second phase that the group proposes. On the other hand, in the scheme, there is no evidence of a third phase, which would be the one referring to the mathematical solution, where the choice of the model is made, therefore, it does not clarify the process that it did for the development of the activity. On the other hand, it did not raise a phase where the need to conclude the mathematical model will be reflected. In addition, it concludes the phenomenon by validating it, but without specifying the model used. In conclusion, the scheme is not entirely related what is proposed by Villa (2009) in his mathematical modeling theory.

Group # 2: The mathematical modeling process proposed by this group was related in a scheme that includes the 5 phases of the mathematical modeling process proposed by Villa (2009) in his theory. It is a scheme created based on the completion of the activity developed in session 1, in addition, it works as a tool for the solution of any mathematical modeling problem raised, since this is designed in a generalized way.

Group # 3: The mathematical modeling process proposed by this group, was based specifically on the development of the activity proposed by the researcher and it does not allow solving any mathematical problem, the phases proposed are specific and unique for the realization of the activity of session # 1. The scheme includes a first phase of data exploration and experimentation which they did the data collection, Villa proposes a second phase of analysis where the construction of the model to be used is done, which is not considered in its scheme as one of the important phases for solving the real problem. They went from phase 1 to phase 3, to solve the problem, and then to conclude the model in a phase 4 and finish in phase 5. Their modeling process contains only 4 phases to solve the problem.

Group # 4: This group presented a 4-phase scheme, which a data collection was carried out in phase 1, followed by an analysis for the construction of the model (phase 2), a deduction of the model for the mathematical solution (phase 3) and the interpretation of the model used to provide a conclusion (phase 4). The proposed scheme does not start with the first thing that a student should do to develop a phenomenon of the real context, which is to explore, experiment, simplify and neither it allows evaluated and validated the phenomenon what they want to develop. That is, this scheme does not match what Villa proposes in his mathematical modeling theory. In addition, they proposed a model only to develop the activity proposed by the researcher and it does not allow the solution of any mathematical problem.

On the other hand, in the third session of the calculus teaching class, the math teachers in training were asked to construct a proposal for a new way of teaching the quadratic function taking into account experimental mathematics practices and mathematics modeling. An evaluation rubric was applied o the results of this proposal, which is designed based on the theories that support this research, in order to contrast what the math teachers in training did with what the theories say, and In addition, a clinical interview was conducted to these, by group, in order to have a holistic analysis of the proposal for the new ways of teaching the quadratic function.

In this respect, when applying the evaluation rubric to the results of the proposal of the new way of teaching the quadratic function carried out by the math teachers in training, the following results were obtained:

Table 2
Results of the evaluation rubric

Aspect to evaluate	Group #1	Group #2	Group #4	Group #5
Statement of the problem in the real world	It allows to identify a real world problem through experimental mathematical practices and helps to carry out an analysis of the behavior of the data obtained by means of a scatter diagram.	It does not allow to identify a real world problem through experimental mathematical practices, it does not help to carry out an analysis of the behavior of the data obtained by means of a scatter diagram.	It allows to identify a real world problem through experimental mathematical practices and helps to carry out an analysis of the behavior of the data obtained by means of a scatter diagram.	It does not allow to identify a real world problem through experimental mathematical practices but it helps to carry out an analysis of the behavior of the data obtained by means of a scatter diagram.
Formulation of the mathematical model	It enables the construction of a mathematical model, but at its discretion, that is, it does not take into account the analysis of data behavior.	It does not allow the construction of a mathematical model that fits the data, taking into account the analysis of their behavior.	It enables the construction of a mathematical model that fits the data, taking into account the analysis of their behavior.	It enables the construction of a mathematical model that fits the data, taking into account the analysis of their behavior.
Results analysis	It facilitates the use of mathematical tools to construct a theoretical solution in order to obtain conclusions from the mathematical model, but it does not allow an analysis of the mathematical model to be carried out.	It does not allow an analysis of the mathematical model and it does not facilitate the use of mathematical tools to build a theoretical solution in order to obtain conclusions from the mathematical model.	It facilitates the use of mathematical tools to construct a theoretical solution in order to obtain conclusions from the mathematical model, but it does not allow an analysis of the mathematical model to be carried out.	It does not allow an analysis of the mathematical model and it does not facilitate the use of mathematical tools to build a theoretical solution in order to obtain conclusions from the mathematical model.
Evaluation of the mathematical model	It facilitates contrasting the conclusions obtained from the mathematical model and the problem itself, in order to allow validation of the mathematical model made.	It does not facilitate to contrast the conclusions obtained from the mathematical model and the problem itself, therefore, it does not allow to validate	It does not facilitate to contrast the conclusions obtained from the mathematical model and the problem itself, therefore, it does not allow to validate the mathematical model carried out.	It does not facilitate to contrast the conclusions obtained from the mathematical model and the problem itself, therefore, it does not allow to validate the mathematical model carried out.

Aspect to evaluate	Group #1	Group #2	Group #4	Group #5
		the mathematical model carried out.		
Refinement of the mathematical model	It does not favor the completion of a new mathematical modeling cycle, even though it is necessary.	It does not favor the completion of a new mathematical modeling cycle, even though it is necessary.	It does not favor the completion of a new mathematical modeling cycle, even though it is necessary.	It does not favor the completion of a new mathematical modeling cycle, even though it is necessary.

Source: self-made

As can be seen in the previous table, the math teachers in training that were taken as a sample, on their average, present difficulties when designing proposals for new ways of teaching quadratic function having experimental mathematics practices and mathematical modeling. In the results of the application of the evaluation rubric, extremes were presented, that is to say, some math teachers in training complied with almost 100% of what the theories say and others, who did not meet even 5% of this.

On the other hand, it can be noted that 100% of the sample, in the proposal of the new way of teaching the quadratic function with experimental mathematics practices and mathematical modeling carried out by math teachers in training, does not favor the realization of a new cycle of mathematical modeling, which is sometimes necessary. Also, it is notorious that not all proposals are clear, that is, that they have inconsistencies either due to lack of information or writing, this is what gives the rationale for the application of the clinical interview. It should be noted that this proposal is a product of the experiences that math teachers had in training at the time of carrying out the activities of session one and two.

As mentioned earlier, a clinical interview was conducted with each work group in order to have a holistic analysis of the results of their proposals, these interviews were recorded with video recordings and subsequently analyzed. The analyzes of the interviews by group were the following:

Group # 1: The experiment done by this group was known with the help of the interview, since, in the proposal presented, it was not clear. The interview showed that the math teachers in training present difficulties in expressing the way in which students identify the variables present in the experiment they are proposing. On the other hand, they did not to show to be clear how the students will build the mathematical model that represents the situation of the experiment to be performed. In addition, math teachers in training, had an error in choosing the experiment, since it was that it generated a quadratic function, and the experiment they proposed, was a linear function. Finally, it was notorious the lack of mastery of the subject presented by math teachers in training, since they were not clear how to handle the situation when the mathematical model built by the students, it is not appropriate to the behavior of the data exposed in the experiment , as Villa puts it, in his mathematical modeling theory.

Group # 2: This group proposed the new way of teaching quadratic function with a problem taken from the real context. On the other hand, it can be noted that in one part of the activity they talk about centimeters and, on the other hand, they talk about meters, the math teachers in training, before this, say that there was a writing error, that the student will not be able to arrive at an answer taking the data with different units of measurement of length, it is notorious, that this does not take into account the conversion that can be made from meters to centimeters or vice versa. In addition, they show that they are not clear how the students will identify the variables present in the experiment. This group did not present appropriation what it is to model mathematically, they did not know what is the correct procedure that must be carried out to build the mathematical model, how it is verified and what actions to take if this model does not adapt to the behavior of the data, as stated by Villa in his mathematical modeling theory.

Group # 3: The proposal of the new way of teaching the quadratic function that this group presented, had a series of questions which evidenced the handling they had about the experimental mathematics practices and in particular, how a phenomenon of the real context is modeled mathematically. They knew what to rely on to build a mathematical model that represented any phenomenon in the real context, how it is verified and what actions should be taken in case the built mathematical model does not adapt to the behavior of the data, as Villa puts it In his theory of mathematical modeling, as well as the usefulness of finding a model that mathematically represents a phenomenon.

Group # 4: The proposal of the new way of teaching the quadratic function presented by this group was devoid of questions, these do not lead students to build the mathematical model that represents the given experiment. It was evident that the teachers in training do not know what is the correct procedure that must be carried out to build a mathematical model, meanwhile, they thinks that this is up to the imagination or dynamics that each student has to do it. In conclusion, it is said that this group of math teachers in training do not handle the mathematical modeling theory proposed by Villa.

4. Conclusions

In the calculus teaching class at the Universidad del Atlántico, three sessions were held, which had an established activity in order to meet the objectives set out in this research. The conclusions obtained after the development of these were:

The built activities allowed to show that mathematical modeling and experimental mathematics practices are fundamental elements in the construction of proposals for new ways of teaching quadratic function, generating in the math teachers in training the interest in seeking new strategies that allow to their pedagogical practices are innovative.

In the calculus teaching class, reflection spaces were created for the math teachers in training to design a problem-solving scheme with mathematical modeling processes, so that with this they presented a proposal for a new way of teaching the quadratic function.

Difficulties were identified in the development of activities by math teachers in training, these were: the use of tenths and hundredths in the sexagesimal system, the misuse of the scales of the Cartesian plane, inappropriate use of graphics for the data analysis and strong tendency to use rule of three without considering its application to linearity. On the other hand, in the design of the proposal for the new way of the quadratic function, math teachers in training, did not take into account the theory of mathematical modeling, this entailed, that in their activities whoever solved them could not build a mathematical model appropriate to the data collected, they could not verify this model and therefore, they could not reconstruct it, in case, if it was necessary, that is, if the mathematical model designed was not consistent with the data.

The impact that there was on the use of experimental mathematics practices and mathematical modeling in math teachers in training in the construction of proposals for new ways of teaching quadratic function was positive, since this gave tools what would allow them to teach in a different way in their pedagogical practice, where knowledge is built by the students who will develop the activities, allowing them to develop competences from experimentation.

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